

# Performance Assessment of DFT-OFDM and DWT-OFDM Systems in the Presence of the HPA Nonlinearity

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*Abstract-This paper investigates performance degradation of conventional Orthogonal Frequency Division Multiplexing (OFDM) and Discrete Wavelet Transform based OFDM (DWT-OFDM) systems when the signals are passed through a nonlinear High Power Amplifier (HPA). In the case of DWT-OFDM, several wavelets such as Daubechies, Symlet and Biorthogonal are evaluated. Simulation results show that DWT-OFDM –specifically Haar (db1)– is more robust against nonlinearity in comparison to DFT-OFDM.*

## I. INTRODUCTION

Multicarrier Modulation (MCM) is an efficient modulation scheme which divides the incoming high rate data into lower rate data. The duration of symbols is increased by simultaneously transmitting  $N$  data symbols which leads to robustness against channels fading, impulsive noise, and Inter Symbol Interference (ISI).

OFDM is a multicarrier scheme commonly used nowadays. OFDM has been widely adopted and standardized across the world. A number of applications and standards which use OFDM include Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), WiFi (IEEE 802.11a/g/j/n), World Wide Interoperability for Microwave Access (WiMAX-IEEE 802.16), Ultra Wide Band Wireless Personal Area Network (UWB Wireless PAN-IEEE 802.15.3a) and Mobile Broadband Wireless Access (MBWA-IEEE802.20). Inverse Fast Fourier Transform (IFFT) and FFT are used in OFDM to multiplex the signals together and demultiplex the signals in the receiver, respectively [1]. A Cyclic Prefix (CP) is prepended to data signals before transmission. The purpose of the CP is to minimize ISI (Inter-symbol interference). However, the CP has disadvantages such as reducing the spectral containment of the channel, power consumption, etc. [2].

Wavelet transformation has recently emerged as a strong candidate for digital communications [3]. In DFT-OFDM systems, signals only overlap in the frequency domain while DWT-OFDM signals overlap both in the time and frequency domains, so there is no need for the CP as in the DFT-OFDM case. Therefore, by using this

transformation, the spectral containment of the channel is improved [4].

Performance of MCM communication systems is highly sensitive to nonlinear distortions arising mainly from the HPA [5-7]. To achieve more output power, transmission power should be increased, which in turn causes the HPA to operate in saturation region. Hence, it seems necessary to assess and compare the DFT-OFDM and DWT-OFDM system performances in the presence of the HPA.

This paper aims to evaluate the impact of the distortion introduced by the nonlinear behavior of a Solid State Power Amplifier (SSPA), as an HPA, which is commonly used in cellular systems. In this study, the Rapp model is used both in DFT-OFDM and DWT-OFDM systems. The Rapp model is characterized by [8]:

$$v_{out} = \frac{v_{in}}{[1 + (\frac{v_{in}}{v_{sat}})^{2p}]^{1/2p}} \quad (1)$$

where  $v_{in}$  is the magnitude of the input signal,  $p$  is smoothness factor,  $v_{out}$  is the magnitude of the output signal, and  $v_{sat}$  is the output saturation level. The smoothness factor controls transition for the amplitude gain as the output amplitude approaches saturation. Fig. 1 shows input-output characteristics for various smoothness factors  $p$ . Also, the phase transfer function is almost zero.

The paper is organized as follows. Section II introduces block diagrams of DFT- OFDM and DWT-OFDM systems, respectively. Section III considers the PAPR (peak to average power ratio) performance in DFT-OFDM and DWT-OFDM

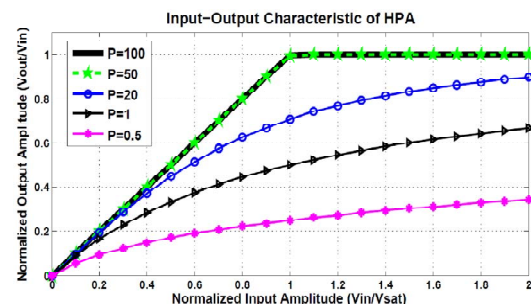


Fig. 1 Input-output characteristic of the Rapp model.

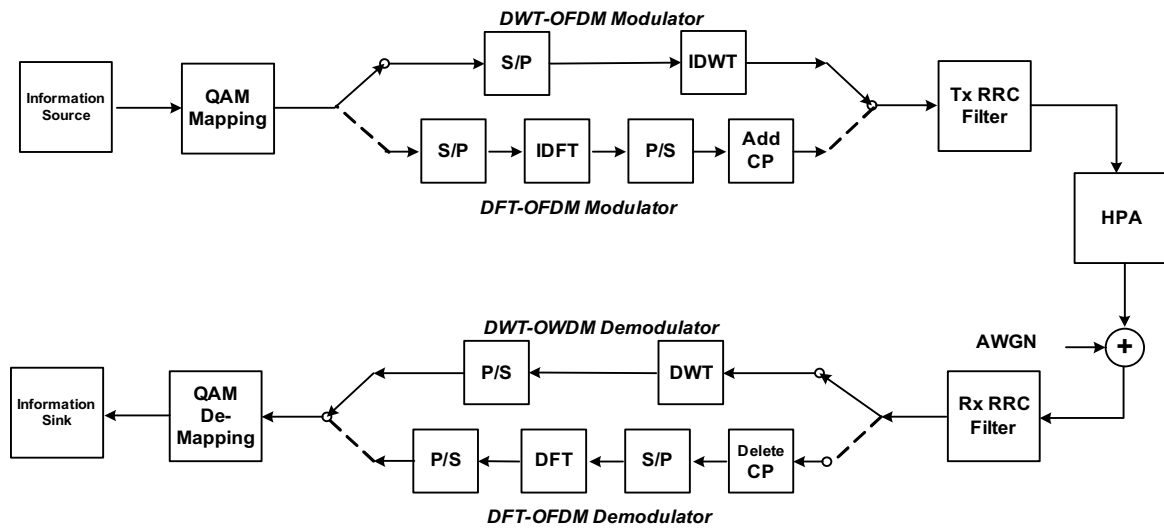


Fig. 2 DFT-OFDM and DWT-OFDM transceiver block diagrams.

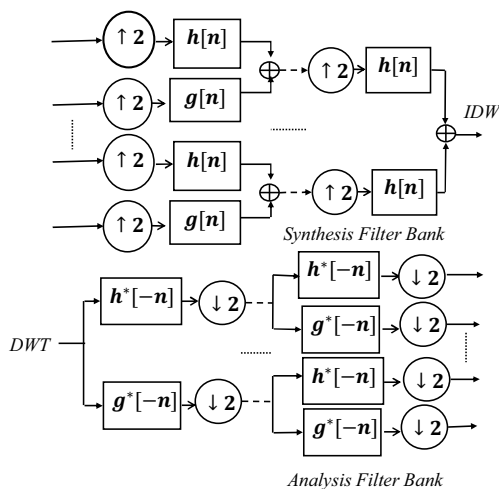


Fig. 3 IDWT and DWT blocks.

systems. Section IV evaluates the Bit Error Rate (BER) performance without SSPA. In section V, the Rapp model is applied to both DFT and DWT MCM systems in order to assess system performances. Finally, section VI concludes the paper.

## II. BLOCK DIAGRAMS OF DFT-OFDM AND DWT-OFDM SYSTEMS

DFT-OFDM and DWT-OFDM transceiver systems are shown in Fig. 2. In DFT-OFDM, the data bit-stream is first mapped onto QAM constellation to form a complex symbol followed by a S/P. Then it is modulated onto orthogonal subcarriers using IDFT. After P/S, a CP (that is 25% of each symbol in practical systems) is wrapped to the symbols. Then the signals are passed through the HPA followed by AWGN channel. At the receiver, the CP is discarded. The resulting signal is demodulated to recover the original data bits.

Wavelet Transform (WT) is a class of generalized Fourier transforms with basis function being localized well both in the time and frequency domains. They are constructed by means of Quadrature Mirror Filter (QMF) pairs [9-10]. It has been shown that DWT-OFDM is more robust to narrowband interference and multipath propagation loss than DFT-OFDM [11]. In DWT-OFDM transmitter, the incoming signal is first converted from serial to parallel. In the case of DWT-OFDM, the number of iterations can be expressed by:

$$\text{Number of iterations} = \log_2(\text{Number of subcarriers}) \quad (2)$$

Fig. 3 shows DWT and inverse DWT (IDWT) blocks. IDWT (as the synthesis filter bank) and DWT (as the analysis filter bank) are used in place of IDFT and DFT, respectively, at the transmitter and receiver. Any iteration of IDWT upsamples two signals and filters one with a High Pass (HP) Finite Impulse Response (FIR) filter and the other one with a Low Pass (LP) FIR filter. The outputs of the HP and LP filters are then subsequently added [12]. Consequently, DWT-OFDM does not require P/S in the transmitter and S/P in the receiver. In our study, several wavelets such as dbN, symN, biorNr,Nd are evaluated. When analysis bank is exchanged with the synthesis bank, the system will be still a perfect reconstruction (PR)[13]. Accordingly, if these wavelets preserve orthogonality between the symbols, it is expected that the Bit Error Rate (BER) plot lies on the theoretical BER plot.

Fundamentally, DFT-OFDM and DWT-OFDM have many similarities as both use orthogonal waveforms as subcarrier. The main difference between DFT-OFDM and DWT-OFDM lies on the shape of the subcarrier and in the way they are created. One important property of wavelet is that the waveforms being used in general are longer than the transform duration of each symbol [14-15]. This causes DWT-OFDM symbols to overlap in the time domain.

The multicarrier symbols of DFT-OFDM do not overlap each other as IDFT and DFT transforms are carried out for each group of subcarriers independently. The use of longer waveforms in DWT-OFDM, on the other hand, allows better frequency localization of subcarriers while in DFT-OFDM the rectangular shape of the DFT window generates large side lobes [16].

Simulation parameters and characteristics of wavelet families are shown in Tables I and II, respectively. For a fair comparison, the CP is not used for DFT-OFDM. Figs. 4a-e illustrate the spectra of 8 adjacent subcarriers for DFT-OFDM and DWT-OFDM with db1, db4, db8 and bior5.5 filters, respectively. By increasing the length of filters in Daubechies family, the bandwidth (BW) of each subcarrier is decreased (Fig. 4-b and Fig. 4-d). The shapes of bior5.5 subcarriers exhibit the unfulfilled orthogonal condition between the HP and LP filters. More details about the system can be seen in [17-18].

### III . PAPR in DFT-OFDM and DWT-OFDM SYSTEMS

One of main drawbacks of OFDM is its high PAPR. Signals with large peaks may be obtained as a result of constructive superposition of subcarriers. PAPR is defined as the ratio between the maximum power occurring in OFDM symbol to the average power of the same OFDM symbol:

$$PAPR = \frac{\max|x(t)|^2}{E[|x(t)|^2]} \quad (3)$$

where  $E[.]$  denotes expectation. PAPR depends linearly on the number of subcarriers, but in systems with a large number of subcarriers, the probability of a symbol with a large PAPR is small and vice versa. This leads to use CDF (Cumulative Distribution Function) to describe PAPR distribution. High peak power is a disadvantage of HPAs. Due to amplifier imperfection, peaks are distorted nonlinearly. The result can be interpreted as an ICI (inter-carrier interference) in the system. In general, PAPR is evaluated from the discrete time samples by oversampling. PAPR can take values in a range that is proportional to the number of subcarriers. In this study, the DFT-OFDM and DWT-OFDM schemes with 64 subcarriers, each modulated with QPSK, were compared in terms of CDF. Fig. 5 shows that while DWT(db1)-OFDM has a comparable PAPR performance, other wavelets exhibit inferior performance in comparison to DFT-OFDM.

### IV. SIMULATION WITHOUT POWER AMPLIFIER

Number of DWT/DFT Point	64
Length of the CP for	0
Data Modulation	QPSK
Channel	AWGN
Shaping Filter	RRC ( $\alpha = 0.22$ , Over sampling Rate = 4)

Name	Length	Vanishing moment	orthogonal
dbN	2N	N	Yes
symN2N	N		Yes
bior3.5	(12.4)	3(dec.)	No
bior5.5	(9.11)	5(dec.)	No

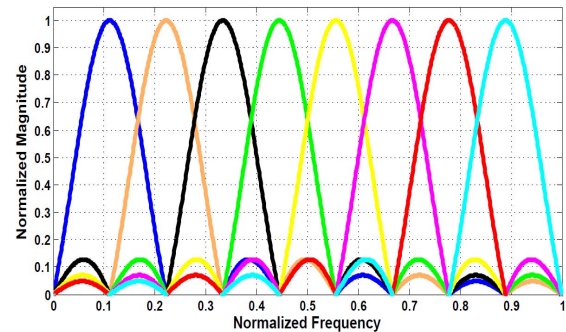


Fig. 4-a Spectra of 8 DFT-OFDM subcarriers.

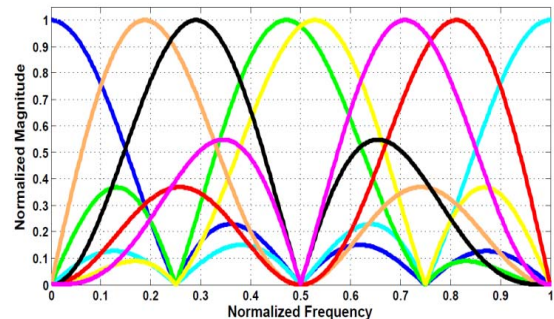


Fig. 4-b Spectra of 8 DWT(db1)-OFDM subcarriers.

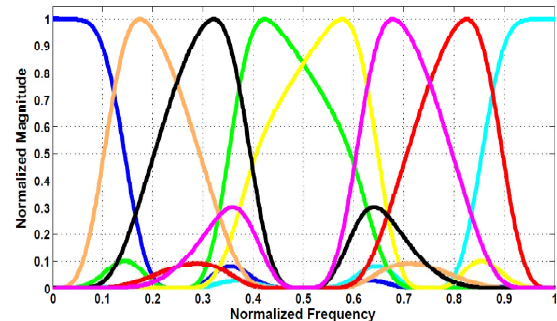


Fig. 4-c Spectra of 8 DWT(db4)-OFDM subcarriers.

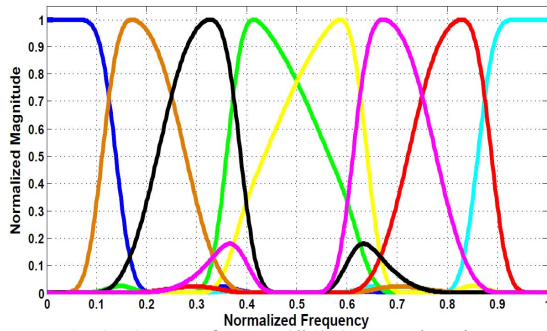


Fig. 4-d Spectra of 8 DWT(db8)-OFDM subcarriers.

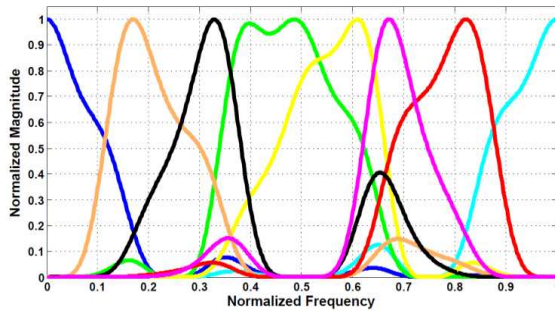


Fig. 4-e Spectra of 8 DWT(bior5.5)-OFDM subcarriers.

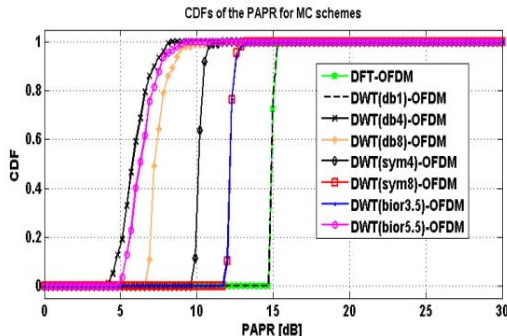


Fig 5 CDFs of the PAPR for different schemes.

As for sanity check, performance of both DFT-OFDM and DWT-OFDM systems without SSPA are evaluated. Fig. 6 shows the BER performance of the QPSK modulation scheme in an AWGN channel. It can be observed in this figure that the BER performances of DWT-OFDM and DFT-OFDM are the same except for bior3.5, and bior5.5. These wavelets are not orthogonal and thus the orthogonality between subcarriers is destroyed. The difference between dbN and symN is not significant, because they do not use any nonlinear element such as HPA and the model is perfectly reconstructive. This validates the simulations.

V. RESULTS IN THE PRESENCE OF THE SSPA

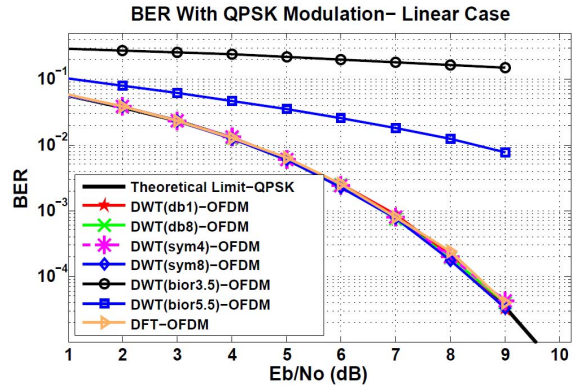


Fig. 6 Performance of DFT-OFDM and DWT-OFDM for the linear case.

An HPA is usually identified by two parameters known as Input Back Off (IBO) and Output Back Off (OBO), defined in decibel as,  $IBO = 10 \log_{10}(\frac{P_{imax}}{P_i})$  and  $OBO = 10 \log_{10}(\frac{P_{omax}}{P_o})$ , respectively, where  $P_i$  and  $P_o$  are the mean power of the input and output signals of the HPA.  $P_{omax}$  is the maximum output power (saturation power), and  $P_{imax}$  is the input power corresponding to the maximum output power [19]. A pictorial description of OBO and IBO is shown in Fig. 7 and defined (on a logarithmic scale) as the difference between the maximum output power and the output power at the quiescent point.

Fig. 8 shows the BER performance of DFT-OFDM and DWT-OFDM when Rapp model is applied with smoothness factor  $p=1$  at  $OBO=3$  dB. In this figure, only db1 outperforms DFT-OFDM. As shown in Fig. 8, by increasing the order of Daubechies and symlet filters, performance of DWT-OFDM system will be degraded. This behavior is more obvious at  $E_b/N_o$  values larger than 12dB.

As another metric, the Total Degradation (TD) parameter is used. The TD is a well-known performance figure used in literature which describes the difference between maximum power of the SSPA and the output power of a linear amplifier required to guarantee a predefined BER [19]. The TD versus OBO to achieve a target BER can be defined as:

$$TD[dB] = (Eb/No_{NL}[dB] - Eb/No_L[dB]) + OBO[dB] \quad (4)$$

Where  $Eb/No_{NL}$  is the required  $E_b/N_o$  in decibels to obtain a target BER for a given value of OBO, and  $Eb/No_L$  is the required  $E_b/N_o$  to obtain the same BER in the absence of nonlinear HPA. The TD depends on the operating point and there is an optimum OBO point of the system corresponding to minimum TD.

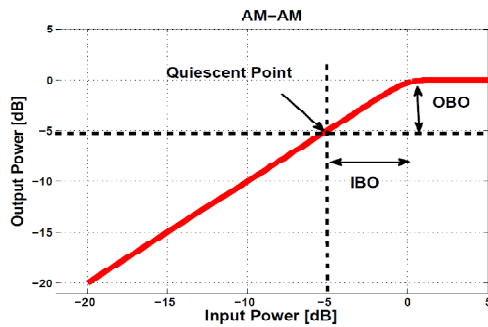


Fig. 7 AM-AM characteristic.

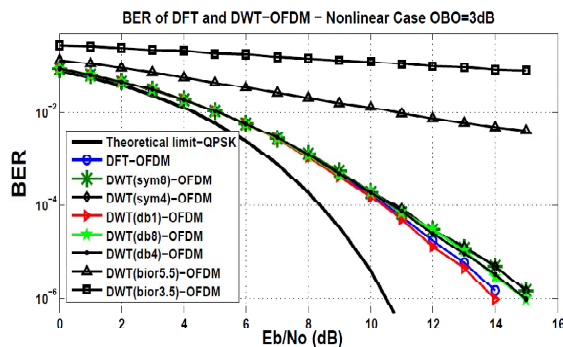


Fig. 8 Performance of DFT-OFDM and DWT-OFDM in the presence of SSPA.

As a consequence, the minimization of the TD can be considered as a fair criterion for the selection of the optimum OBO value.

Figs. 9, 10, and 11 illustrate the TD performance of DFT-OFDM and DWT-OFDM with Daubechies, Symlet and Biorthogonal families, respectively, at target BER =  $10^{-4}$ . In Fig. 9 only DWT(db1)-OFDM shows a superior TD performance over DFT-OFDM. The difference between TD performance of DWT(db1)-OFDM and DFT-OFDM at OBO= 1dB is approximately 0.5 dB. Also, it should be emphasized that by increasing OBO (higher degree of linearity) the TD values of both systems become the same. It means that there is no difference in BER of two systems for the linear scenario. By increasing the length of the filters in Daubechies family, the performance of DWT-OFDM is degraded. Due to time-overlap of DWT-OFDM signals, when the filter length is increased, the data length is increased and hence it dose not fit anymore in the available space. So, a part of the samples has to be discarded. In Fig. 10, DFT-OFDM outperforms DWT-OFDM and by increasing the filter length in Symlet family, the TD performance of DWT-OFDM is degraded. The performance of DWT-OFDM with Biorthogonal family is not comparable to DFT-OFDM (Fig. 11). The above results were obtained with no equalization. In order to fairly compare different MC schemes, other simulations were carried out using the block-type

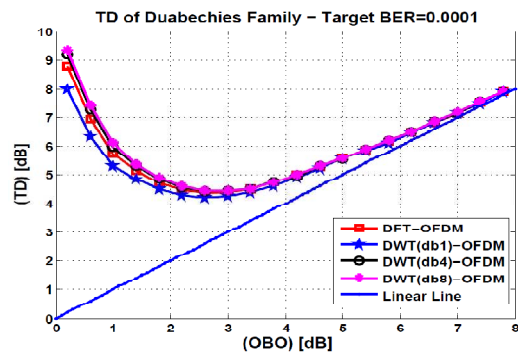


Fig. 9 Performance of DFT-OFDM and DWT-OFDM with Daubechies Family.

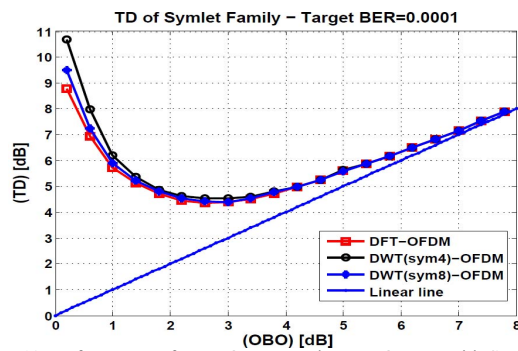


Fig. 10 Performance of DFT-OFDM and DWT-OFDM with Symlet Family.

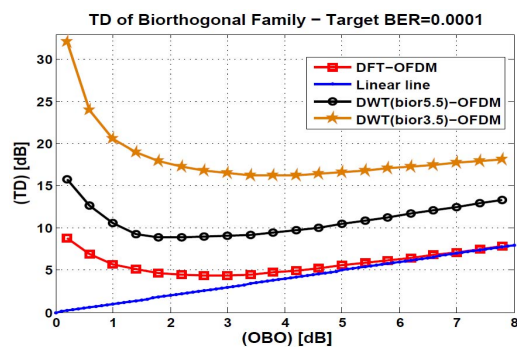


Fig. 11 Performance of DFT-OFDM and DWT-OFDM with Bi-orthogonal Family.

(time-multiplexed) pilot-aided channel estimation. This is due to this fact that the block-type pilot arrangement has been developed under the assumption of slow fading channel, while the other basic arrangement i.e. the comb-type (frequency-multiplexed) has been introduced for equalizing the fast fading channels [20]. In this study, the information signal is impaired by the time-invariant (nonlinear) behavior of the HPA rather than its temporal variations. In other words, its characteristics remain constant over a packet duration. For a slow time-varying channel, the block-type pilots estimate the channel more efficiently than the comb-type pilots since an entire symbol is used as pilot carriers at the

start of each packet [20]. Fig. 12 shows BER performances of DFT-OFDM and DWT-OFDM at OBO = 1dB using two consecutive OFDM symbols (and then averaging) as pilots. As this figure shows only DWT(db1)-OFDM outperforms DFT-OFDM.

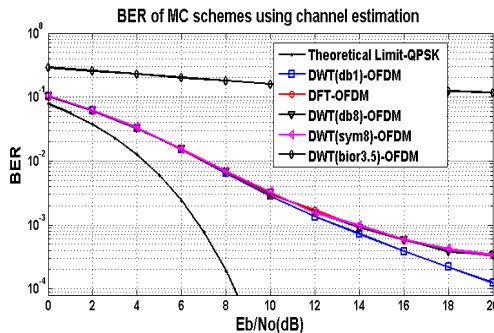


Fig. 12 BER of DFT-OFDM and DWT-OFDM using block-type estimation.

## VI. CONCLUSIONS

In this paper the BER, PAPR, and TD performances of DFT-OFDM and DWT-OFDM in the presence of SSPA -as an HPA- were evaluated using Rapp model. According to simulation results, it was found that the BER performance of DWT-OFDM is the same as DFT-OFDM in AWGN channel for the linear system i.e. without SSPA as a nonlinear block. As the filter length was altered among the members of the same family (except for the biorthogonal family), no perceivable difference was observed in the system performance. However, if Rapp model is applied, this difference becomes significant. In Daubechies and Symlet families, when the length of the filter was increased, the BER and TD performances were degraded. The result showed that just db1 (Haar) wavelet for the DWT-OFDM system achieved better BER and TD performances compared to DFT-OFDM. The above results were confirmed for the corresponding equalized schemes as well. Also, some DWT-OFDM schemes showed superior PAPR performances than that DFT-OFDM.

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